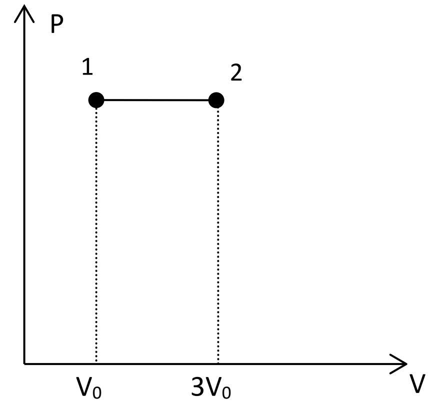
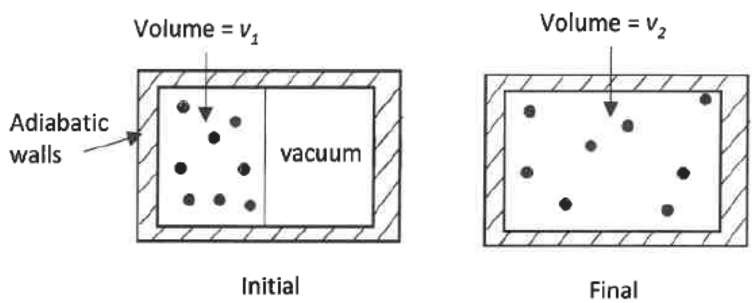
Statistical Mechanics Qualifying Exam, Spring 2022

Short Problem 1:

Under a reversible isobaric (P=const) expansion, the volume of a system of ideal gas is changed from V_0 to $3V_0$. Find the change of entropy ΔS_{12}.



Short Problem 2:



Starting from the First Law of Thermodynamics, show that the specific change in entropy of an ideal gas that undergoes a free expansion in a container with adiabatic walls is:

$$\Delta s = R ln\left(\frac{v_2}{v_1}\right)$$

where R is the ideal gas constant and v_2 and v_1 are the final and initial volumes, respectively. What characteristic of the ideal gas allows you to arrive at the expression above? Given that the walls of the container are adiabatic and that it is a free expansion, i.e. P = 0, what assumption is made in calculating the change in entropy?

Long problem 1:

Consider an extreme relativistic gas contained in volume V in equilibrium at temperature T. The gas is composed of N distinguishable particles so that single-particle energy and momentum are related by $\varepsilon=pc$, where c is the speed of light. The single-particle energy state in the range of p to $(p+dp)$ is $4\pi V p^2 dp/h^3$.

(a) Find the equation of state of the gas;

(b) Calculate the internal energy U;

(c) Compare U with that of the ordinary monoatomic ideal gas.

Long problem 2:

Consider a system of N noninterating simple harmonic oscillators (SHO) of frequency ω in equilibrium with a thermal reservoir of temperature T. The energy of each SHO is $\varepsilon = n\hbar\omega$, where $n = 0, 1, 2, 3, \ldots\infty$, $\hbar$ is the Planck constant.

(a) Calculate the partition function of the system;
(b) Obtain the Helmholtz free energy;
(c) Obtain the entropy of the system;
(d) Calculate the internal (or total) energy.