Short Problem 1:

Consider the observables $A = x^2$ and $B = L_z$:

a) Construct the uncertainty principle for $\sigma_A \sigma_B$

b) Evaluate σ_B for the hydrogen state ψ_{nlm}

c) What can you conclude about $\langle xy \rangle$ in this state?

Short Problem 2:

Consider a three-dimensional vector space spanned by an orthonormal basis $|1\rangle$,$|2\rangle$,$|3\rangle$. Kets $|\alpha\rangle$ and $|\beta\rangle$ are given by:

$$|\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle, \qquad |\beta\rangle = i|1\rangle + 2|3\rangle \tag{1}$$

a) Construct $\langle\alpha|$ and $\langle\beta|$ in terms of the dual basis $\langle 1|$,$\langle 2|$,$\langle 3|$

b) Find $\langle\alpha|\beta\rangle$ and $\langle\beta|\alpha\rangle$

c) Find all nine matrix elements of the operator $\hat{A} = |\alpha\rangle\langle\beta|$ in that basis and construct the matrix $\mathbf{A}$. Is it Hermitian?

Long Problem 1:

a) Evaluate the following commutators:
 i) $[L,\ S.L]$
 ii) $[L.S,\ S]$
 iii) $[L,\ S.J]$
 iv) $[L.S,\ S^2]$
 v) $[L.S,\ J^2]$

b) The Hamiltonian for an electron in the Hydrogen atom with spin-orbit coupling has the form:

$$H = H_0 + \alpha L.S \tag{2}$$

Where α is small. The eigenstates of the unperturbed Hamiltonian H_0 can be expressed in terms of angular momentum quantum numbers $|njlsm_j\rangle$. Calculate the first order energy perturbation of each state from the spin-orbit coupling.

c) Show that m_l and m_s are no longer good quantum numbers when the perturbation is included, but m_j is.

Long Problem 2:

Find the energy levels and wave functions of the ground state and the first excited state for a system of two non-interacting identical spin-1/2 particles moving in a common external harmonic oscillator potential.