Short Problem 1:

A beam of electrons is passed through a (Stern- Gerlach) filter which allows only electrons with spin along the +*z* direction to pass. These electrons then pass through a second filter that only allows electrons with spin along +*x* direction pass. What fraction of the electrons that enter the second filter pass through.

Short Problem 2:

Given a three-dimensional Hilbert space, there are two observables A and B that, with respect to the basis $|1\rangle, |2\rangle, |3\rangle$, and are represented by the matrices

$$A = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix}; B = \begin{pmatrix} 0 & B & 0 \\ B^* & 0 & 0 \\ 0 & 0 & B_3 \end{pmatrix} \tag{1}$$

Verify that the two observables are compatible

(see next page)

Long Problem 1:

A symmetrical system with moments of inertia $I_x = I_y$ and I_z in the bodies axes frame is described by the Hamiltonian

$$H = \frac{1}{2I_x}\left(L_x^2 + L_y^2\right) + \frac{1}{2I_z}L_z^2 \tag{3}$$

where the moments of inertia I_x, I_y, I_z are parameters and not operators.

a) Calculate the eigenvalues and eigenstates of the Hamiltonian

b) What values are expected for a measurement of $\hat{B} = L_x + L_y + L_z$ for any state?

Hint: Recall that the angular momentum operators can be written in terms of raising and lowering operators

$$L_x = \frac{L_+ + L_-}{2}, L_y = \frac{L_+ - L_-}{2i} \tag{4}$$

and that the raising and lowering operators follow the relationship

$$L_+|l,m\rangle = \hbar\sqrt{l(l+1) - m(m+1}|l,m+1\rangle$$

$$L_-|l,m\rangle = \hbar\sqrt{l(l+1) - m(m-1}|l,m-1\rangle$$

c) The system begins at $t = 0$ in the state $|l = 3, m = 0\rangle$, what is the probability that a measurement of L_x at $t = 4\pi I_x/\hbar$ will yield the value $\hbar$.

(see next page)

Long Problem 2:

We have a quantum system with two levels, $|0\rangle$, $|1\rangle$, with hamiltonian

$$H = -\frac{1}{2}\hbar\omega(|0\rangle\langle 0| - |1\rangle\langle 1|)$$

- Are the states $|0\rangle$, $|1\rangle$ stationary states? which are their energies?
- Let's consider the operator $a = |0\rangle\langle 1|$ and its hermitian conjugate $a^\dagger = |1\rangle\langle 0|$. Show:

$$\{a, a^\dagger\} = aa^\dagger + a^\dagger a = 1; a^2 = a^{\dagger 2} = 0$$

- Which are the eigenvalues of the operator $N = aa^\dagger$?
- Let's consider the hermitian operator $A = a + a^\dagger$. Which are its eigenstates and eigenvalues?
- Let's imagine the system is in an initial state in the eigenstate of the operator A with eigenvalue 1. Which is the uncertainty of A (ΔA) at time t?