Short Problem 1

Starting with the Maxwell's equations, show that the wave equations for **E** and **B** in free space are:

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

(useful identities: $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$, $\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B}$)

Short Problem 2

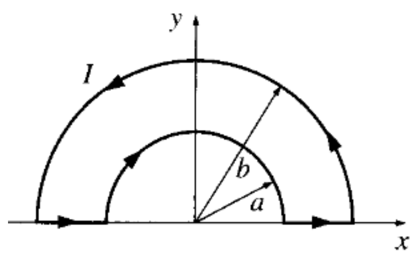
A charge distribution that is spherically symmetric but not uniform radially produces an electric field of magnitude $E=Kr^4$, directed radially outward from the center of the sphere. Here r is the radial distance from that center, and K is a constant. What is the volume density ρ of the charge distribution?

Long Problem 1

A piece of wire bent into a loop, as shown in the figure, carries a current that increases linearly with time:

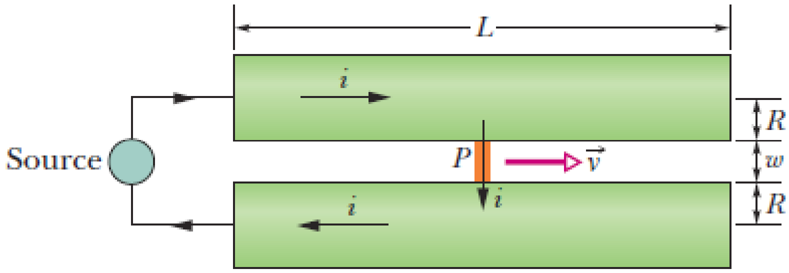
I(t) = kt.

1. Calculate the retarded vector potential A at the origin.
2. Find the electric field at the center.
3. Explain why this neutral wire (total charge density is zero) can produce an electric field?
4. Can you determine the magnetic field from the vector potential (A) expression you deduce in part (1)? Why?



Long Problem 2

The figure below is an idealized schematic drawing of a rail gun. Projectile *P* sits between two wide rails of circular cross section; a source of current sends current through the rails and through the (conducting) projectile (a fuse is not used). Let *w* be the distance between the rails, *R* the radius of each rail, and *i* the current.



1. Show that the force on the projectile is directed to the right along the rails and is given approximately by

$$F = \frac{i^2\mu_0}{2\pi}\ln\frac{w + R}{R}.$$

2. If the projectile starts from the left end of the rails at rest, find the speed v at which it is expelled at the right.