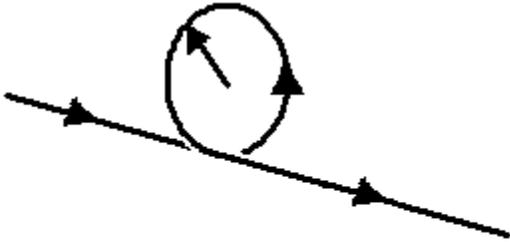


Short Problem 1:

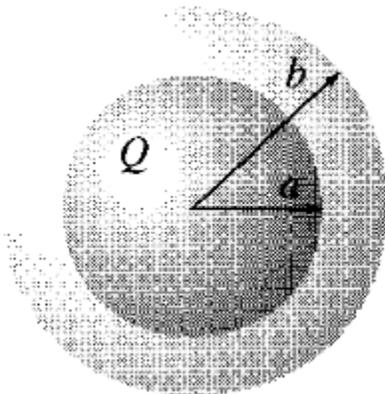
An infinitely long straight wire is modified such that it includes a circular loop whose plane is perpendicular to the direction of the (otherwise long, straight) wire. The wire is insulated, and carries a current I , and the radius of the circular loop is R . Find the magnitude and direction of the magnetic field \mathbf{B} at the center of the loop.



Short Problem 2:

A spherical conductor, of radius a , carries a charge Q . It is surrounded by linear dielectric material of susceptibility χ_e , out of radius b . Find the energy of this configuration and compare it with case without the dielectrics.

- Calculate electric displacement (\mathbf{D}) in the air part, the dielectric part and the conductor using Gauss's law in the presence of Dielectrics.
- Calculate the electric field (\mathbf{E}).
- Calculate the total energy and compare it with case without the dielectric (increase or decrease)



Long Problem 1:

An electromagnetic wave is incident from the left side of an interface between two different media. As a result, two electromagnetic waves travel away from the interface (see the diagram below). Assume that all the waves are traveling in either the $\pm x$ directions and that the interface is in the y - z plane ($x = 0$).

Assume that the electric field of the incident wave is given by:

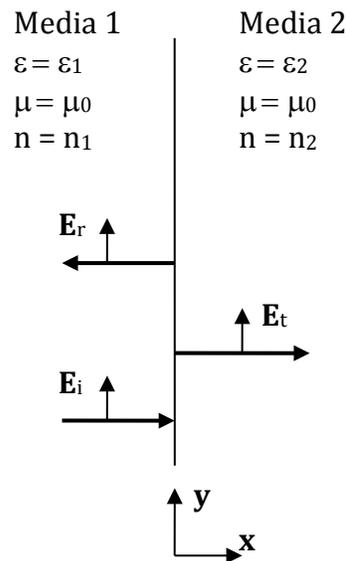
$$\mathbf{E}_i = A_0 \hat{y} e^{i(k_1 x - \omega t)}$$

Assume that the electric fields of the two waves going away from the interface are given by:

$$\mathbf{E}_r = A_r \hat{y} e^{i(-k_1 x - \omega t)}$$

$$\mathbf{E}_t = A_t \hat{y} e^{i(k_2 x - \omega t)}$$

Where $k_1 = \omega(\mu_0 \epsilon_1)^{1/2}$ and $k_2 = \omega(\mu_0 \epsilon_2)^{1/2}$ and \hat{y} is the unit vector in the y direction.



- Find the magnitude and direction of the magnetic fields (\mathbf{H}_i , \mathbf{H}_r , \mathbf{H}_t) associated with the three waves in terms of the electric fields (\mathbf{E}_i , \mathbf{E}_r , \mathbf{E}_t) given above.
- Use the boundary conditions that the tangential component of the total \mathbf{E} field is continuous at the boundary and the tangential component of the total \mathbf{H} field is continuous at the boundary to find the amplitudes A_r and A_t in terms of A_0 , ϵ_1 and ϵ_2 (or A_0 , n_1 and n_2).
- Find the net, time-averaged, power per unit area crossing the boundary in terms of A_0 , ϵ_1 and ϵ_2 (or in terms A_0 , n_1 and n_2).

Long problem 2:

The current I flows down a resistor as shown in the figure. The potential difference between the ends is V and the length of the wire is L . what is the total power flowing into it? Compare the result from Joule heating law.

- (a) Assuming the electric field is uniform, what is the electric field parallel to the wire?
(b) Use the Ampere's law to calculate the magnetic field at the surface ($r=a$) of the wire (

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I,$$

- (c) Calculate the Poynting flux flowing into the wire through the circumferential surface (

$$\mathbf{S} \equiv \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}).$$

- (d) Calculate the energy per unit time passing in through the circumferential surface (

$\int \mathbf{S} \cdot d\mathbf{a}$) and compare it with the energy dissipated in the resistor through Joule heating law.

