Classical Mechanics Qualifying Exam, Spring 2022

Short Problem 1

A block is projected along a frictionless surface with an initial velocity v_0. The block is subject to a velocity dependent air resistance given by $\vec{F}(\vec{v}) = -c_1\vec{v}$ where c_1 is a constant.

a) Find the position of the block as a function of time

b) Show that the furthest distance the block will go is given by:

$$x_{max} = \frac{mv_0}{c_1} \tag{1}$$

Short Problem 2

An object sits on a disk rotating with angular velocity ω (see Fig. 1). The disk is parallel to the ground and it has a coefficient of friction = 0.5. At what radius does an object of mass m start to slide?

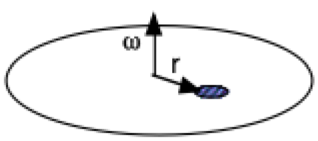


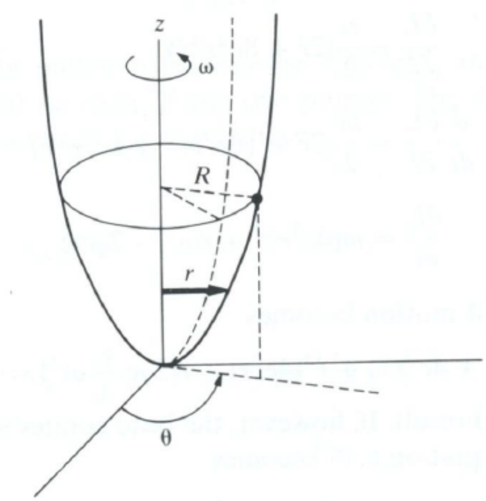
Fig. 1

Long Problem 1

A bead slides along a smooth wire bent in a shape of a parabola $z = ar^2$. The wire is rotating about its vertical symmetry axis with angular veocity ω.

(a) Construct the Lagrangian equation of motion (in terms of r). Assume potential energy $U = 0$ at $z = 0$.

(b) The bead moved around a circle of radius R. What is the value of a?



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Long Problem 2

In certain situations, particularly in one-dimensional systems, it is possible to incorporate frictional effects without introducing the dissipation function. As an example consider the following Lagrangian:

$$L = e^{2\gamma t}\left[\frac{m\dot{q}^2}{2} - \frac{kq^2}{2}\right]$$

a) Find the equations of motion. How would you describe the system? Are there any constants of motion?

b) Suppose a point transformation of the form $s = qe^{\gamma t}$. What is the transformed Lagrangian in terms of s?

c) Find the equation of motion for the transformed Lagrangian.